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STATUS OF GLUEBALL MASS CALCULATIONS IN LATTICE GAUGE THEORY*

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Abstract

The status of glueball spectrum calculations in lattice gauge theory is briefly reviewed, with focus on the comparison between Monte Carlo simulations and small-volume analytical calculations in SU(3). The agreement gives confidence that the large-volume Monte Carlo results are accurate, at least in the context of the pure gauge theory. An overview of some of the technical questions, which is aimed at non-experts, serves as an introduction.

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1 INTRODUCTION

Glueballs are basic predictions of quantum chromodynamics (QCD). Combining almost any ideas on color confinement with the observation that gluons carry color leads to the conjecture of color singlet states formed out of the glue—the "glue balls." On the other hand, the existence of glueballs is inconclusive experimentally. Typical glueball candidates include $f_0(991)$ [1], $\eta(1440)$, $f_2(1720)$ and the G resonances of the GAMS experiment [2]. For each state it is probably safe to say that its "true believers" do not yet out-number its "frowning skeptics." Hence, the subject of glueballs is one that could well benefit from better and more reliable theoretical guidance.

Glueball mass calculations in lattice gauge theory are not (yet) more reliable than other calculations. The foremost sense in which the the lattice calculations might be "better" is that they are done using the first principles of QCD. Unfortunately, the calculations summarized here leave out some of the first principles: in particular the interactions of gluons with quarks are neglected. Nevertheless, the glueball mass computation in pure SU(N) gauge theory is a necessary step in the development of numerical simulations of lattice gauge theory. Until lattice gauge theorists can provide a theoretically correct result for the pure gauge theory, it is unlikely that they will provide a correct result for full QCD.

The presentation starts with a brief technical review, aimed at non-experts. We shall discuss the framework, paying attention to systematic details. We hope that this approach better prepares the reader for interpreting realistically results in lattice gauge theory. We then present a compilation of results for scalar and tensor, $PC = ++$ glueballs in pure SU(3) gauge theory.

2 GAUGE THEORY ON THE LATTICE

Lattice gauge theory is the only generally applicable scheme for non-perturbative calculations in quantum field theory. The ultraviolet divergencies are regulated by replacing (Euclidean) space-time with a discrete lattice. For computer simulations the "space-time" is usually a torus— a finite box with periodic boundary conditions

$$R^4 \rightarrow T^4 \rightarrow N_S^3 \times N_T \text{ lattice.} \tag{1}$$

Writing a for the lattice spacing, $L = N_S a$ is the physical size of the spatial volume, and, as always in the Euclidean formalism, the time extent $T = N_T a$ is related to the physical temperature by $\Theta = 1/T$.

To maintain gauge invariance the path-integration variables are no longer components of the gauge potential $A_\mu^a(x)$, but instead parallel transporters along the links connecting two adjacent lattice sites:

$$U_\mu(x) = \mathrm{P}\exp\int_x^{x+a\hat{\mu}} A_\mu(y)dy_\mu. \tag{2}$$

The gauge field interacts according to some action S with the "naive continuum limit"

$$\lim_{a\rightarrow 0} S = \int d^4x\,\mathcal{L}, \quad \mathcal{L} = \frac{1}{4g_0^2}\sum_a (F_{\mu\nu}^a)^2. \tag{3}$$

Usually the Wilson plaquette action is used.

In lattice field theory a sets the natural scale of all dimensionful quantities. If m is a mass, in MeV, then the lattice calculation can only determine the product ma. Mass ratios $(m_1 a/m_2 a) = m_1/m_2$ are the natural predictions, and one must then choose one mass to set the scale before comparing with experiment. This is not cheating, because the Lagrangian $\mathcal{L}$ has one free parameter, the bare coupling g_0. The trade-off g_0 for m is analogous to eliminating the bare coupling e_0 of QED in favor of the fine structure constant $\alpha = 1/137$.

On the torus the rotational symmetry is reduced to cubic symmetry. (The hypercubic lattice also breaks the symmetry.) Hence the states are classified by cubic symmetry quantum numbers, rather than spin. The cubic group has only five irreducible representations, denoted by A_1, A_2, E, T_1 and T_2, with dimensions 1, 1, 2, 3, and 3, respectively. For large enough L (and small enough a) one expects restoration of rotational symmetry, which is signaled by "accidental" degeneracies of cubic multiplets. For example, a doublet in the representation E must combine with a triplet in the representation T_2 to form the quintet of spin $J = 2$. To emphasize the need to realize rotational symmetry restoration, we shall use the cubic labels: A_1 for the scalar, and E and T_2 for the two states that ought to form the tensor.

3 NUMERICAL WORK

In numerical work one generates an ensemble of configurations $\{U_\mu(x)\}^{(n)}$, $n = 1, \ldots, N_{\text{conf}}$, distributed with weight e^{-S}. The path integral for a correlation function is estimated by

$$C_r(t) = \langle \Phi_r^*(t)\, \Phi_r(0) \rangle \approx \frac{1}{N_{\text{conf}}} \sum_n \Phi_r^*(t; \{U_\mu(x)\}^{(n)})\, \Phi_r(0; \{U_\mu(x)\}^{(n)}) \tag{4}$$

where Φ_r is an interpolating field operator for states with quantum numbers denoted by r. Eq. (4) expresses Monte Carlo integration with importance sampling of the path integral. At large Euclidean times t the correlation function takes the form

$$C_r(t) \approx \exp(-m_{1,r} t). \tag{5}$$

Fitting eq. (5) yields the lowest mass $m_{1,r}$ in the channel with r quantum numbers.

Before meaningful numbers can be extracted, one must take several limits. We will discuss these limits in the order needed for a rigorous definition of the continuum quantum field theory.

1. Take $N_{\text{conf}} \to \infty$; in this limit the Monte Carlo integration becomes exact. The right-hand-side of eq. (4) yields the correct result for the *lattice* theory.

2. Take $a \to 0$; this is the continuum limit, but it must be approached with $L = N_S a$ and $T = N_T a$ held fixed. Hence, the lattice size parameters N_S and N_T must increase like $1/a$.

3. Take $N_T a = T \to \infty$, the zero temperature limit.

4. Take $N_S a = L \to \infty$, the infinite volume limit.

If carried out, Limit 1 would require an infinite amount of CPU time; Limits 2-4 would require an infinite amount of memory and an infinite amount of CPU time to process it. In practice we neither hope nor need to carry out any of these limits. The price paid is an uncertainty in the estimate of a physical quantity. The uncertainty from Limit 1 is *statistical*, because it decreases as $1/\sqrt{N_{\text{conf}}}$. The statistical error can also be reduced by variance reduction techniques [3]. The uncertainties of Limits 2-4 are *systematic*. They are best controlled by systematic study of a, T or L dependence, perhaps using extrapolation.

Here are some rules of thumb indicating when each kind of error has become tolerable:

1. N_{conf}: As in all Monte Carlo integration the acceptable statistical error depends on the circumstances. In the context of lattice gauge theory a commendable criterion is to have statistical error bars small enough to analyze the systematic effects.

2. a: When mass ratios m_1a/m_2a are a-independent for fixed L we have some confidence in the results; such behavior is called "scaling." The perturbative renormalization group predicts
$$ma = \left(\frac{2N}{g_0^2}\right)^{(\beta_2/\beta_1^2)} \exp\left(\frac{1}{2\beta_1 g_0^2}\right), \tag{6}$$
where the β_i are the first two coefficients of the Callan-Symanzik β-function. Such "asymptotic scaling" behavior (would) inspire supreme confidence in the lattice results.

3. T: The temperature should be low enough to resolve different states with the same quantum numbers:
$$\Theta \ll m_{2,r} - m_{1,r}. \tag{7}$$

4. L: When mass ratios are L-independent for fixed a optimists maintain that the large L limit has been reached. Pessimists will fit to asymptotic formulae such as
$$m(\infty) = m(L)\left[1 + c_0 \exp\left(-\frac{\sqrt{3}}{2}Lm\right)\right] \tag{8}$$
for masses [4] or
$$K(\infty) = K(L) + \frac{\pi}{3L^2} \tag{9}$$
for the ('t Hooft) string tension [5]. Realists will continue to act like optimists until they acquire the computer time to act like pessimists.

To disentangle the various effects one should look for methods with optimal signal-to-noise ratios. For glueball masses this means choosing Φ_r sensibly [3]. Once the (statistical) error bars are small, one should study the systematic effects by forming dimensionless ratios, for which a drops out.

4 RESULTS FROM SU(3)

Fig. 1 is compilation of for several groups' results for the SU(3) gauge group. It is an update of the figure presented in ref. [16], using $z_{\sqrt{K}} = L\sqrt{K} = N_S a\sqrt{K}$ as the measure of the volume. The scatter in the data points arises not only from statistical fluctuations, but also from variations in the lattice spacing: roughly speaking 0.12 fm $\gtrsim a \gtrsim$ 0.05 fm.

There are two good reasons to measure the volume by $z_{\sqrt{K}}$ rather than $z_{A_1^{++}} = Lm_{A_1^{++}} = N_S a m_{A_1^{++}}$. The statistical fluctuations of $\sqrt{K}$ are smaller than those of $m_{A_1^{++}}$, as emphasized in ref. [12]. Also, the A_1^{++} mass is likely significantly suppressed by lattice artifacts [17] for the Wilson action at larger lattice spacings. Since fig. 1 attempts to disentangle statistical fluctuations from systematic effects of lattice spacing and volume, $z_{\sqrt{K}}$ seems most suitable.

The curves are the result of analytical calculations [14] like those of refs. [15]. These calculations start with Lüscher's perturbatively derived effective Hamiltonian for the zero-momentum modes of the gauge field [18], but the spectrum is obtained non-perturbatively. The agreement is impressive in the region where both calculational schemes are valid. In SU(2) the agreement is similar, and the discrepancy is known to be caused by non-zero lattice spacing effects [19]. The curves in fig. 1 are significant because they argue for the correctness of the Monte Carlo mass ratios for small $z_{\sqrt{K}}$, inspiring confidence at larger $z_{\sqrt{K}}$.

There are several striking features of the results. First, the ratio $\sqrt{K}/m_{A_1^{++}}$ is surprisingly constant for $z_{\sqrt{K}} > 0.6$. Second, for $0.2 < z_{\sqrt{K}} < 2.0$ the two multiplets that should form the tensor glueball are not at all degenerate. But in the region $1.8 < z_{\sqrt{K}} < 2.8$ the mass of the E representation changes by a factor of two and for $z_{\sqrt{K}} > 3$ the $m_{E^{++}}$ and $m_{T_2^{++}}$ agree within statistical errors. The crossover region is not in a surprising place, $L \approx 1$ fm, but it is intriguing that $m_{E^{++}}$ behaves so differently from $\sqrt{K}$, $m_{A_1^{++}}$ and $m_{T_2^{++}}$.

Taking $z_{\sqrt{K}} > 2.6$ as close enough to the infinite volume, we find averages of $\sqrt{K}/m_{0^{++}} = 0.308 \pm 0.020$ and $m_{2^{++}}/m_{0^{++}} = 1.543 \pm 0.082$. The subscripts now refer to spin, because the degenerate E and T_2 masses suggest restoration of rotational symmetry. Setting the scale with $\sqrt{K} = 420$ MeV gives predictions of $m_{0^{++}} = 1370 \pm 90$ MeV and $m_{2^{++}} = 2115 \pm 125$ MeV. These values are tantalizingly close to resonances reported by GAMS [2], but it would be imprudent to draw exciting conclusions. While fig. 1 indicates the systematic effects of non-zero a and finite L are under control, it says nothing about the systematic effect of neglecting quarks. In addition to mentioning obvious effects such as mixing and decay, one might comment that the string tension K is not an especially natural quantity to use for setting the scale in full QCD.

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References

[1] K.L. Au, D. Morgan and M.R. Pennington, *Phys. Lett.* **B167** (1986) 229; *Phys. Rev.* **D35** (1987) 1633

[2] D. Alde, et al, *Z. Phys.* **C43** (1989) 541; *Phys. Lett.* **B216** (1989) 447

[3] F. Brandstaeter, A.S. Kronfeld and G. Schierholz, Fermilab/HLRZ preprint FERMILAB-PUB-89/199-T, HLRZ 89-72

[4] M. Lüscher, in *Progress in Gauge Field Theory*, G. 't Hooft et al, eds., (Plenum, New York, 1984); *Comm. Math. Phys.* **104** (1986) 277

[5] R.D. Pisarski and O. Alvarez, *Phys. Rev.* **D26** (1982) 3735;
Ph. de Forcrand, G. Schierholz, H. Schneider and M. Teper, *Phys. Lett.* **B160** (1985) 137

[6] C. Vohwinkel and B.A. Berg, *Phys. Rev.* **D40** (1989) 584

[7] T.A. DeGrand, *Phys. Rev.* **D36** (1987) 3522

[8] C. Michael and M. Teper, *Phys. Lett.* **B206** (1988) 299; *Nucl. Phys.* **B314** (1989) 347

[9] Ape Collaboration: M. Albanese, et al, *Phys. Lett.* **B192** (1987) 163; **B205** (1988) 535

[10] Ape Collaboration: M. Albanese, et al, *Phys. Lett.* **B197** (1987) 400;
L.A. Fernandez and E. Marinari, *Nucl. Phys.* **B295[FS21]** (1988) 51;
E. Marinari, private communication ($\beta = 6$, $N_S = 18$ results)

[11] G. Schierholz, *Nucl. Phys.* **B**(*Proc. Suppl.*)**9** (1989) 244;
F. Brandstaeter, A.S. Kronfeld, K.J.M. Moriarty and G. Schierholz, in preparation

[12] J. Hoek, Rutherford Laboratory preprint RAL-89-081

[13] A. Patel, private communication

[14] C. Vohwinkel, Florida State University Ph. D. Thesis; to be published

[15] J. Koller and P. van Baal, *Phys. Rev. Lett.* **58** (1987) 2511; *Nucl. Phys.* **B302** (1988) 1; *Nucl. Phys.* **B**(*Proc. Suppl.*)**4** (1988) 47

[16] P. van Baal and A.S. Kronfeld, *Nucl. Phys.* **B**(*Proc. Suppl.*)**9** (1989) 227

[17] A. Patel, R. Gupta, G. Guralnik, G.W. Kilcup and S.R. Sharpe *Phys. Rev. Lett.* **57** (1986) 1288

[18] M. Lüscher, *Phys. Lett.* **B118** (1982) 391; *Nucl. Phys.* **B219** (1983) 233

[19] P. van Baal, *Phys. Lett.* **B224** (1989) 397

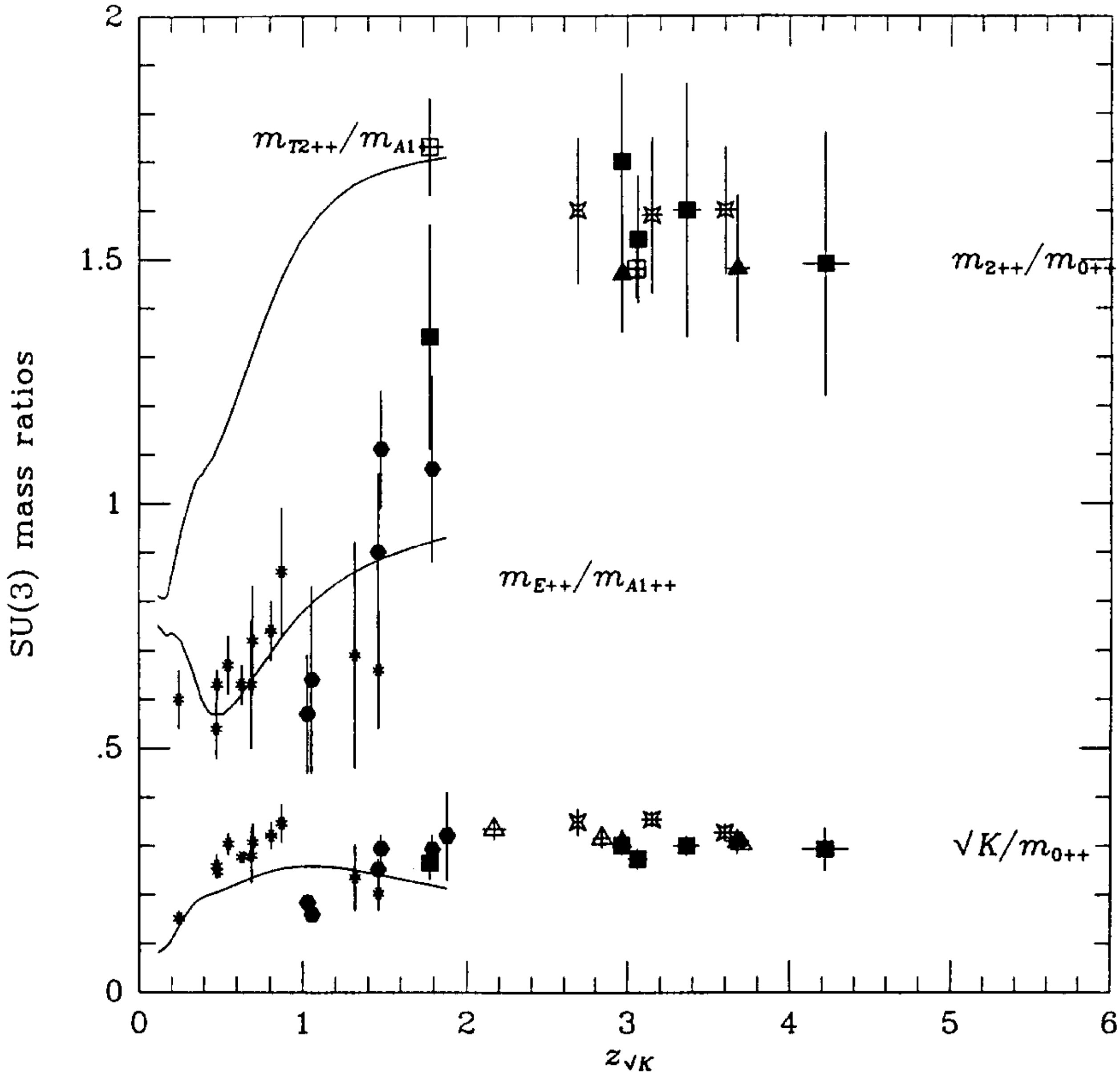

Figure 1: *Plot of dimensionless ratios* $\sqrt{K}/m_{A_1^{++}}$, $m_{E^{++}}/m_{A_1^{++}}$ *and* $m_{T_2^{++}}/m_{A_1^{++}}$ *vs.* $z_{\sqrt{K}} = L\sqrt{K} = N_S a\sqrt{K}$. *The symbols are asterisks from ref. [6], a solid circle from ref. [7], open (T_2) and closed (K and E) squares from ref. [8], open triangles from ref. [9], closed triangles from ref. [10], four-pointed stars from ref. [11], hexagons from ref. [12], and the open circle (which used a MCRG improved action) from ref. [13]. The curves are analytic results [14] valid in small volumes [15].*